The self-blindable U-Prove scheme from FC'14 is forgeable

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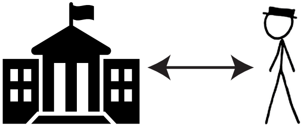
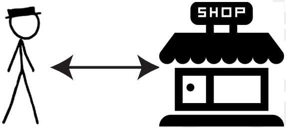
Credential schemes

IssueCredential



ShowCredential



Features:

- No communication with issuer during transactions
- Provably unforgeable

Attribute-based credential schemes

$\Longrightarrow$

Credential

- Attributes $(x_1, \ldots, x_n)$
- Signature on attributes

Features:

- Selective disclosure
- **Unlinkability**

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Existing ABCs

Idemix Camenisch, Lysyanskaya	RSA-like groups	unlinkable	unforgeable
U-Prove Brands	elliptic curves	not unlinkable	?
FC'14 scheme Hanzlik, Klukzniak	**elliptic curves**	**unlinkable**	**forgeable**

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FC'14 scheme Hanzlik, Klukzniak	**elliptic curves**	**unlinkable**	**forgeable**

Hanzlik and Kluczniak's scheme

Setup:

- Type 2 bilinear pairing $e\colon G_1 \times G_2 \to G_T$
  - G_1, G_2 elliptic curves of prime order
- Issuer public key: $(e, g_0, \ldots, g_n, p, p', p_0, p_1)$
  - with $g_0, \ldots, g_n \in G_1, \quad p, p' \in G_2, \quad p_0 = (p')^z, p_1 = p^f$
- Issuer private key: (f, z)

A credential over attributes $(x_1, \ldots, x_n)$:

$$(\underbrace{h, h_2, h_3, h_4,}_{\in G_1} \underbrace{\alpha, b_1, b_2}_{\text{numbers}})$$

- $h = (g_0 g_1^{x_1} \cdots g_n^{x_n})^\alpha$
- $h_2 = h^f$
- $h_3 = h^{b_1} h_2^{b_2}$
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Showing a credential

Credential: $(x_1, \ldots, x_n), (h, h_2, h_3, h_4, \ \alpha, b_1, b_2)$

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Showing a credential, disclosing x_1:

- Blind the credential as above
- Zero-knowledge proof over $x_2, \ldots, x_n, \alpha k, b_1 \ell, b_2 \ell$

Showing a credential

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The attack: 2 colliding users

Set $\widetilde{g}_i = g_i^f$

- $h_j = (g_0 g_1^{x_{1,j}} \cdots g_n^{x_{n,j}})^\alpha, \quad h_{2,j} = h^f$
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User 1 and 2 calculate:

$$\frac{h_{3,1}^{1/d_{1,1}}}{h_{3,2}^{1/d_{1,2}}} = g_0^u \widetilde{g}_0^{\,v} g_1^w$$

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- $\widetilde{g}_1$ shared across all credentials
- Group order known $\Rightarrow$ can invert exponents

The attack: many colliding users

$$h_3 = g_0^{c_0} \widetilde{g}_0^{d_0} g_1^{c_1} \widetilde{g}_1^{d_1}, \qquad h_4 = h_3^z$$

- 2 users can remove $\widetilde{g}_1$ from h_3 and $\widetilde{g}_1^z$ from h_4
- 8 users can:
  - compute $\widetilde{g}_0$ and $\widetilde{g}_0^z$, g_0^z, $\widetilde{g}_1$, $\widetilde{g}_1^z$, g_1^z
  - compute signatures on base points g_i
  - create credentials containing arbitrary attributes.
- n attributes $\Rightarrow 2^{2n+1}$ users $\Rightarrow 2n+2$ users

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Ingredients of attack:

- Fixed base points $g_0, \ldots, g_n$
- Invertibility of exponents

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