An efficient self-blindable attribute-based credential scheme

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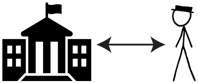
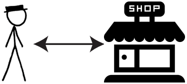
April 3, 2017
Financial Crypto, Malta

Attribute-based credential schemes

IssueCredential



ShowCredential



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- Attributes $(k_1, \ldots, k_n)$
- Signature on attributes

$\Longleftrightarrow$

- Selective disclosure
- Efficient
- Unforgeable
- Unlinkable

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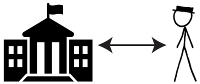

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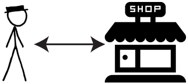

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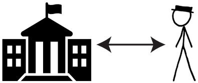

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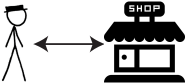

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- Bilinear pairing $e\colon G_1 \times G_2 \to G_T$ of prime order p
  - i.e. $e(P^a, Q) = e(P, Q^a) = e(P, Q)^a$ for all $P \in G_1,\ Q \in G_2$
- Issuer secret key: $a, a_1, \ldots, a_n, z \in_R \mathbb{Z}_p := \mathbb{Z}/p\mathbb{Z}$

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- $e(K^a, Q) = e(S, Q) \stackrel{?}{=} e(K, A) = e(K, Q^a)$
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Showing a blinded credential $n = 3$

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Choose $\alpha, \beta \in_R \mathbb{Z}_p^*$

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2: show credential, disclosing k_2, k_3

- Send $(\bar{K}, \bar{S}, \bar{S}_1, \bar{S}_1, \bar{S}_2, \bar{S}_3)$, $(\tilde{C}, \tilde{T})$
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- Prove knowledge of β, κ, k_1 s.t.

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Security

Theorem (Unlinkability)

The ShowCredential protocol is a zero-knowledge proof of knowledge for possession of a credential containing the disclosed attributes.

Theorem

Credentials are forgeable $\iff \exists$ *algorithm that can output tuples*

$$(\kappa, K, S, T) \quad \text{with} \quad S = K^a, \; T = (KS^\kappa)^z$$

with differing κ*'s, without knowing* a, z*, in polynomial-time.*

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Taking the LRSW assumption, our credential scheme is unforgeable.

Lysyanksaya, Rivest, Sahai, Wolf '99

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Known Exponent Assumption

Given G_1 and $P, P^a \in G_1$, the only way to output R, R^a is to take $r \in \mathbb{Z}_p$ and output $(P^r, (P^a)^r) \Rightarrow$ implies LRSW assumption.

Performance

Comparison of our scheme (254 bits) with Idemix (IRMA project, `www.irmacard.org`, 1024 bits $\rightsquigarrow$ 144 bits)

# attributes		prover		verifier	
total	discl.	This work	Idemix	This work	Idemix
6	1	2.9	11.7	5.7	11.2
9	1	3.4	14.3	8.0	14.0
12	1	4.2	17.1	10.2	16.9
6	5	2.1	7.6	5.9	9.2
9	8	2.4	7.4	7.9	10.7
12	11	2.8	7.5	9.9	12.0